

Short answer questions.  
Define the following:

01. Truth-preserving: *An inference rule is truth-preserving iff, given true premises, it always produces true conclusions.*

02. Derived line: *A derived line is a line in a proof other than the premises. Derived lines are produced by applying inference rules to either premises or other derived lines.*

03. Justification: *Each derived line requires a justification, i.e. a citation of the inference rule(s) and other line(s) from which the derived line is derived.*

Answer the following as succinctly as you can:

04. Explain why the rule of addition is truth-preserving. *The rule of addition is truth-preserving because of the particular truth function of disjunction. The rule of addition states that given some true statement A we may infer  $(A \vee B)$ , where B is any statement whatsoever. The truth function for disjunction stipulates that when one of two disjuncts is true then the entire disjunction will be true, regardless of the truth value of the other disjunct. So the rule of addition is truth-preserving because, given a premise which is true, any concluded disjunction which uses that premise as one of its disjuncts must also be true.*

05. Does it matter, for our purposes of proof construction, whether the rules of inference are falsity-preserving? Are they falsity-preserving? *Call an inference rule "falsity-preserving" iff every time it operates on false premises it produces a false conclusion. It does not matter, given our use of the inference rules so far, whether the inference rules are falsity-preserving. We are merely interested in the fact that our inference rules are truth-preserving, i.e. their operation on true premises will always yield true conclusions. As a matter of fact, not all of our inference rules are falsity preserving. Good examples of inference rules which are not falsity preserving are Addition and Simplification. This can be proved nicely with a simple truth-table.*

Proofs.

Complete the following proofs using the eight inference rules.

[Note: There are a variety of legal ways to construct good proofs. The examples I provide below are *not* the only correct answers. An equally good, error-free proof, might be quite different from those below. Are you able to construct an error-free proof which is shorter than one of the ones I have provided? If so, kudos!]

06-13. Easy.

06. 01.  $A \vee B$   
 02.  $\sim B \vee S$   
 03.  $\sim A$  /  $S$   
 04.  $B$  1,3 *DS*  
 05.  $\sim\sim B$  4 *DN*  
 06.  $S$  2,5 *DS*
07. 01.  $X \supset Y$   
 02.  $\sim Z \supset \sim Y$   
 03.  $X$  /  $Z$   
 04.  $Y$  1,3 *MP*  
 05.  $\sim\sim Y$  4 *DN*  
 06.  $\sim\sim Z$  2,5 *MT*  
 07.  $Z$  6 *DN*
08. 01.  $D \cdot E$   
 02.  $G \cdot H$   
 03.  $G \vee E$   
 04.  $(G \cdot E) \supset J$  /  $J$   
 05.  $E$  1 *simp*  
 06.  $G$  2 *simp*  
 07.  $G \cdot E$  5,6 *conj*  
 08.  $J$  4,7 *MP*
09. 01.  $N \supset M$   
 02.  $K$   
 03.  $K \supset (\sim M \cdot L)$  /  $\sim N$   
 04.  $\sim M \cdot L$  2,3 *MP*  
 05.  $\sim M$  4 *simp*  
 06.  $\sim N$  1,5 *MT*
10. 01.  $P \supset R$   
 02.  $(P \supset Q) \supset S$   
 03.  $R \supset Q$  /  $S$   
 04.  $P \supset Q$  1,3 *HS*  
 05.  $S$  2,4 *MP*
11. 01.  $\sim S \supset (R \vee W)$   
 02.  $\sim R \cdot \sim S$  /  $W$   
 03.  $\sim S$  2 *simp*  
 04.  $R \vee W$  1,3 *MP*  
 05.  $\sim R$  2 *simp*  
 06.  $W$  4,5 *DS*
12. 01.  $\sim A$

02.  $A \vee (B \wedge \sim C)$   
 03.  $C$  /  $\sim B$   
 04.  $B \wedge \sim C$  1,2 *DS*  
 05.  $\sim \sim C$  3 *DN*  
 06.  $\sim B$  4,5 *MT*
13. 01.  $P \cdot \sim Q$   
 02.  $(\sim R \wedge T) \vee Q$   
 03.  $P \wedge (S \wedge \sim R)$  /  $S \wedge T$   
 04.  $\sim Q$  1 *simp*  
 05.  $\sim R \wedge T$  2,4 *DS*  
 06.  $P$  1 *simp*  
 07.  $S \wedge \sim R$  3,6 *MP*  
 08.  $S \wedge T$  5,7 *HS*
- 14-18. Medium.
14. 01.  $\{(M \cdot N) \vee O\} \supset R$   
 02.  $O \cdot M$   
 03.  $O \wedge N$   
 04.  $O \vee S$  /  $R \vee S$   
 05.  $O$  2 *simp*  
 06.  $(M \cdot N) \vee O$  5,6 *add*  
 07.  $R$  1,6 *MP*  
 08.  $R \vee S$  7 *add*
15. 01.  $R \supset \sim(U \vee W)$   
 02.  $U \cdot H$   
 03.  $(U \vee W) \wedge (T \wedge R)$  /  $U \cdot \sim T$   
 04.  $U$  2 *simp*  
 05.  $U \vee W$  4 *add*  
 06.  $T \wedge R$  3,5 *MP*  
 07.  $\sim \sim(U \vee W)$  5 *DN*  
 08.  $\sim R$  1,7 *MT*  
 09.  $\sim T$  6,8 *MT*  
 10.  $U \cdot \sim T$  4,9 *conj*
16. 01.  $H \supset I$   
 02.  $(E \wedge G) \cdot (G \vee J)$   
 03.  $(I \vee E) \cdot (H \vee E)$   
 04.  $(G \vee \sim K) \cdot (I \vee \sim L)$  /  $I \vee G$   
 05.  $H \vee E$  3 *simp*  
 06.  $E \wedge G$  2 *simp*  
 07.  $I \vee G$  1,5,6 *Dil*

17. 01.  $(\sim K \cdot \sim Q) \{(\sim R \sim B) \cdot (\sim S K)\}$   
 02.  $\sim S \vee \sim R$   
 03.  $K \supset Q$   
 04.  $I \cdot \sim Q \quad / \quad \sim B$   
 05.  $\sim Q \quad 4 \text{ simp}$   
 06.  $\sim K \quad 3,5 \text{ MT}$   
 07.  $\sim K \cdot \sim Q \quad 5,6 \text{ conj}$   
 08.  $(\sim R \sim B) \cdot (\sim S K) \quad 1,7 \text{ MP}$   
 09.  $\sim R \sim B \quad 8 \text{ simp}$   
 10.  $\sim S K \quad 8 \text{ simp}$   
 11.  $\sim \sim S \quad 6,10 \text{ MT}$   
 12.  $\sim R \quad 2,11 \text{ DS}$   
 13.  $\sim B \quad 9,12 \text{ MP}$

18. 01.  $(L \supset D) \cdot \sim D$   
 02.  $\sim L \supset (G \supset H)$   
 03.  $(\sim L \vee K) \supset \sim H \quad / \quad \sim G$   
 04.  $L \supset D \quad 1 \text{ simp}$   
 05.  $\sim D \quad 1 \text{ simp}$   
 06.  $\sim L \quad 4,5 \text{ MT}$   
 07.  $\sim L \vee K \quad 6 \text{ add}$   
 08.  $\sim H \quad 3,7 \text{ MP}$   
 09.  $G \supset H \quad 2,6 \text{ MP}$   
 10.  $\sim G \quad 8,9 \text{ MT}$

19-21. Hard.

19. 01.  $(P \vee R) \cdot \sim P$   
 02.  $(R \vee C) \supset (P \supset K)$   
 03.  $(\sim P \vee D) \supset (R \supset L) \quad / \quad K \vee L$   
 04.  $\sim P \quad 1 \text{ simp}$   
 05.  $P \vee R \quad 1 \text{ simp}$   
 06.  $R \quad 4,5 \text{ DS}$   
 07.  $R \vee C \quad 6 \text{ add}$   
 08.  $\sim P \vee D \quad 4 \text{ add}$   
 09.  $P \supset K \quad 2,7 \text{ MP}$   
 10.  $R \supset L \quad 3,8 \text{ MP}$   
 11.  $P \vee R \quad 6 \text{ add}$   
 12.  $K \vee L \quad 9,10,11 \text{ Dil}$

20. 01.  $\sim Z \supset (O \supset X)$   
 02.  $R \cdot \sim V$   
 03.  $(S \supset \sim W) \cdot (R \supset \sim Z)$   
 04.  $(\sim Z \vee \sim Y) \supset (\sim X \vee W)$   
 05.  $(R \vee P) \supset (S \vee V) \quad / \quad \sim O$

06. $R$	$2 \text{ simp}$
07. $R \sim Z$	$3 \text{ simp}$
08. $\sim Z$	$6,7 \text{ MP}$
09. $O \ X$	$1,8 \text{ MP}$
10. $\sim Z \vee \sim Y$	$8 \text{ add}$
11. $\sim X \vee W$	$4,10 \text{ MP}$
12. $S \sim W$	$3 \text{ simp}$
13. $R \vee P$	$6 \text{ add}$
14. $S \vee V$	$5,13 \text{ MP}$
15. $\sim V$	$2 \text{ simp}$
16. $S$	$14, 15 \text{ DS}$
17. $\sim W$	$12,16 \text{ MP}$
18. $\sim X$	$11,17 \text{ DS}$
19. $\sim O$	$9, 18 \text{ MT}$

21.	01. $(M \vee R) \ (D \vee B)$	
	02. $(M \vee G) \ (\sim D \ C)$	
	03. $(B \cdot C) \ (D \vee \sim A)$	
	04. $H \cdot \sim D$	
	05. $D \vee M$	$/ \ \sim A$
	06. $\sim D$	$4 \text{ simp}$
	07. $M$	$5,6 \text{ DS}$
	08. $M \vee R$	$7 \text{ add}$
	09. $M \vee G$	$7 \text{ add}$
	10. $D \vee B$	$1, 8 \text{ MP}$
	11. $\sim D \ C$	$2,9 \text{ MP}$
	12. $B$	$6,10 \text{ DS}$
	13. $C$	$6, 11 \text{ MP}$
	14. $B \cdot C$	$12,13 \text{ conj}$
	15. $D \vee \sim A$	$3,14 \text{ MP}$
	16. $\sim A$	$6,15 \text{ DS}$